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**Abstract**

A life cycle model is constructed and used to examine how a farmer's incentive to invest in environmental capital changes over time and in response to changes in various structural parameters such as the productivity of regular farm capital, the scale objective of the farmer, the rate of interest on farm debt, an upper limit on the allowable debt to asset ratio, and the speed of debt repayment. Simulation results for the special case of quadratic utility are presented along with a set of general theoretical results. A number of counterintuitive outcomes emerge such as a farmer invests more in environmental capital with a higher interest rate on farm debt and invests less when the farm is given more access to credit. Conventional subsidies that raise farm income are shown to lower a farmer's investment in environmental capital for "early" career farmers and raise investment for "late" career farmers. Overall the model should be useful for policy makers who are designing environmental programs or evaluating the unintended impacts of standard farm income stabilization programs.



Keywords: farm, investment, environment, life cycle.

JEL classification: D91, D92, Q12.

# A Life Cycle Model of Farm Investment in Environmental Capital

## 1 Introduction

The purpose of this paper is to develop a life cycle model of farm investment in environmental capital, which is assumed to include environment-related equipment, buildings, technologies and various other forms of environmental best management practices. The model is used to examine how a farmer's incentive to invest changes over his or her life cycle and depends on factors such as the productivity of regular farm capital, the farm's financial characteristics and farmer specific characteristics such as his or her rate of discount and attitude toward the environment. The dynamic interaction between consumption, investment in productive and environmental capital, and farm debt gives rise to unique results, several of which are counterintuitive. For example, a higher cost of capital does not necessarily reduce investment in environmental capital, and higher farm income, possible the result of a conventional subsidy, can raise raise or lower investment, depending on the farmer's particular stage of the farming life cycle. More liberal lending policies tend to reduce investment, as does a larger target farm size and faster required repayment of farm debt. Much of this paper is devoted to explaining the reason for the various comparative static results.

An important reason for developing a model of farm investment in environmental capital is to assist in the design of environmental farm programs and to critique standard farm income stabilization programs. Many countries now place considerable weight on environmental outcomes when implementing farm programs. For example, the EU has a long standing tradition of using cross compliance programs to ensure that farmers who received decoupled payments adhere to pre-specified environmental standards [Brady, Kellermann, Sahrbacher, and Jelinek, 2009, Mosnier, Ridier, Kephaliacos, and Carpy-Goulard, 2009]. Similar programs in the U.S. are referred to as conservation compliance [Hoag and Holloway, 1991, Govindasamy and Cochran, 1995, 1997, Gardner, Hardie, and Parks, 2010]. Other popular programs target input reduction such as pesticides [Falconer and Hodge, 2001] and fertilizer [Tanaka and Wu, 2004], or share in the cost of farm level adoption of environmental technologies [Weerksink, McKitrick, and Nailor, 2001, Dupont, 2010]. The results of this analysis, which highlight how a farmer's incentive to invest changes over time and in response to changes in various structural parameters, is expected to be useful during the design phase of environmental programs.

Standard farm programs that are designed to stabilize farm incomes are often criticized for their lack of attentiveness to environmental outcomes. For example, there has been considerable debate amongst agricultural economists whether crop insurance and related risk management programs induce farmers to apply more fertilizer and pesticides to their land [Babcock, Fraser, and Lekakis, 2003]. One of the goals of the current analysis is to shed light on how investment in environmental capital depends on farm programs that raise average farm income. Although the current model is limited in its capacity to critique farm income stabilization programs because it is non-stochastic, the key result that higher farm income has offsetting impacts on a farmer's incentive to invest in environmental capital is nevertheless important. Higher income raises the opportunity cost of diverting resources away from productive farm capital for "early" career farmers but simultaneously reduces the marginal utility of consumption and therefore raises the incentive to invest in environmental capital for "late" career farmers. These incentives change continually through out the farmer's life cycle, but in the simulation results the overall impact of higher farm income on a farmer's incentive to invest in environmental capital is negative.

There is a highly diverse literature that address the issue of incentives for farmers to invest in environmental capital. For example, as discussed above there has been considerable interest in the link between risk reduction and farming intensity. Other papers have argued that better access to credit or a reduction in the reliance on credit may also have an environmental impact because of higher farming intensity [Vercammen, 2007]) There is a very large literature on the adoption of best management practices by farmers [Knowler and Bradshaw, 2007, Prokopy, Floress, Klotthor-Weinkauf, and Baumgart-Getz, 2008] and a smaller literature on targeted programs and optimal design of environmental farm programs in the presence of asymmetric information [Wu and Babcock, 1995, Bontems, Rotillon, and Turpin, 2005, Giannakas and Kaplan, 2005, Feng, 2007, Fraser, 2009]. The incentives for farmers to participate in carbon offset markets has also been examined [Fulton and Vercammen, 2009]).

Despite this larger and growing literature, there has no attempts to use a farm life cycle model to rigorously examine a farmer's incentive to invest in environmental capital or to examine the optimal design of an environmental farm program in a life cycle framework. There is a large literature that utilize dynamic models of consumption and agricultural investment, but much of this literature is empirical and tends to focuses on uncertainty, risk aversion, adjustment costs and the impact of lump sum payments on investment incentives [Sckokai and Moro, 2009]. There are also many life cycle models that have been used to examine a farmer's incentive to invest in productive capital (e.g.,Phimister [1993], Phimister [1995]). This paper demonstrates that a farmer's incentive to invest in environmental capital is significantly different that the incentive to invest in productive capital or the incentive to consume. Hence, a special type of analysis that incorporates investment in environmental capital into a life cycle model is warranted.

So what exactly is a farming life cycle? Although there is no one accepted definition, for the purpose of this paper a farming life cycle is defined as the set of consumption and investment decisions that take place over a farmer's full planning horizon. An important feature of a life cycle is the farmer's access to credit. Upper limits on a farmer's debt to asset ratio and the terms of debt repayment will have important implications for the extent that a farmer is willing to invest in environmental capital. The results of this analysis clearly demonstrate that the incentive for a farmer to invest in environmental capital changes continually over time and in response to changes in the farmer's operating environment. Hence, it should not be surprising that the large empirical literature which aims to identify the determinants of farm level adoption of environmental best management practices has yielded very few consistent results [Knowler and Bradshaw, 2007, Prokopy et al., 2008].

The next section discusses the general features of the model and the standard approaches in the literature that are used to examine regular farm investment and consumption incentives. For the current analysis the life cycle model is divided into two main phases that are examined in Sections 3 and 4, respectively. Simulation results are presented in Section 5. The last section of the paper contains a summary and concluding comments.

# 2 Model Overview

This section has three parts. In the first part some general assumptions of the model are laid out. Part 2 is used to discuss the standard modeling approaches. The last section describes the specific assumptions which are required to implement the chosen modeling approach.

## 2.1 General Assumptions

A representative farmer with a $T$ period time horizon optimally chooses consumption and investment in productive and environmental capital.[1]The farmer makes recurring decisions involving investment and consumption in order to maximize a discounted stream of utility. Each period the farmer receives utility from regular consumption and from the farm's stock of environmental capital. To keep the analysis simple only a special type of environmental capital is considered. Specifically, the stock of environmental capital provides utility for the farmer, but it does not affect the productive capacity of the farm. An example of such capital is manure storage with an impermeable pad and odour reduction devices. A second example is riparian buffer strips that are designed to reduce the flow of soil nutrients into waterways that run through the farmer's land. A final example is a pesticide applicator fitted with low-drift nozzles that are used to minimize spray drift damage in neighboring fields.[2]

The farmer receives utility from the stock of environmental capital because of both personal environmental benefits (e.g., reduced odour from manure, cleaner surface water and safer sources of ground water for drinking) and indirect environmental benefits (e.g., the feeling of being a responsible stewart of the land and a "good" neighbor to the rural and urban residents who live on surrounding properties). Investment in environmental capital by the farmer also

[1]It is common in the life cycle literature to assume that the farmer has a bequest motive to eliminate the incentive for the farmer to gradually dispose of farm assets as time $T$ approaches [Phimister, 1993]. The current analysis is kept simple by assuming a very large value for $T$ such that the effects of capital stock disposition as time $T$ approaches are not important due to discounting.

[2]Suppose a farmer chooses to convert farmland into wetland habitat for ducks. In this model the cost of investing in the wetlands would include the cost of renting additional farmland to ensure that the investment in environmental capital does not affect the productive capacity of the farm.

generates public benefits that are not internalized by the farmer. These public benefits are not included the model because the current analysis focuses solely on farm level decision making in the absence of farm environmental programs.

Let $C_t$ denote the monetary value of consumption in period $t$ that is financed with farm revenue. A negative value for $C_t$ implies that an off-farm source of income is being used to subsidize the farming operation.[3]Let $U(C_t)$ denote consumption-dependent utility, which accrues to the farmer at the end of period $t$. Without loss in generality, utility from consuming off-farm income is not made explicit in the utility function. Let $E_t$ denote the stock of environmental capital at the beginning of period $t$. Specifically, assume that $E_t$ is the deviation of the stock in environmental capital above an exogenous best management practice benchmark. Hence, a negative value of $E_t$ implies that the stock of environmental capital is positive but it is below the benchmark. Let $V(E_{t+1})$ denote the level of utility that flows to the farmer at the end of period $t$ assuming that the stock of environmental capital that is carried out of period $t$ and into period $t+1$ is equal to $E_{t+1}$ units.

The analysis is simplified by assuming that the two sources of utility within a particular period are additively separable, and also that each particular source of utility is additively separable over time.[4] Let $W$ denote the period 1 present value of combined utility for the farmer that will result if personal consumption follows the sequence $C_1$, $C_2$, etc. and the stock of farm environmental capital follows the sequence $E_2$, $E_3$, etc. The relevant expression is

$$W = \sum_{t=1}^{\infty}(1+\rho)^{-t}\left(U(C_t) + V(E_{t+1})\right) \tag{1}$$

Within equation (1) the parameter $\rho$ is the farmer's rate of discount.

Let $K_t$ denote the stock of productive capital on the farm at the beginning of period $t$. Let $f(K_t)$ denote net farm income that is generated by the farm's productive capital in period $t$. Net

[3]Suppose the farmer earns $\bar{C}$ per period from off-farm income. If $C_t > 0$ then all of the off-farm income is consumed and total consumption is $C_t + \bar{C}$. If $C_t < 0$ then $-C_t$ units of off-farm income is used to make up the shortfall in farm investment and loan interest expenditures and the rest of off-farm income is consumed. Assume that $\bar{C}$ is sufficiently large to ensure that $\bar{C}+C_t$ does not fall below minimum required consumption for the farmer.

[4]These are strong assumptions because it is reasonable to assume that a farmer's marginal utility of consumption is higher when the stock of environmental capital is high and vice versa.

farm income is an increasing function of farm capital, which implies $f'(K_t) > 0$. Additional restrictions on the form of $f(K_t)$ are discussed below. The state equation for productive farm capital is given by $K_{t+1} = K_t + I_t^K$ where $I_t^K$ represent the level of investment in productive capital. Similarly, the state equation for environmental capital is $E_{t+1} = E_t + I_t^E$ with $I_t^E$ representing the level of investment in environmental capital. The model is kept simple by assuming zero stock depreciation for both types of capital.

The third state equation in the model is for outstanding farm debt, $D_t$. Assumptions are made below which ensure optimized farm debt takes on a strictly positive value throughout the farmer's life cycle. These assumptions are reasonable because without a bequest motive the farmer has no incentive to save wealth beyond period $T$. With $r$ denoting the per-period rate of interest on debt, the appropriate state equation can be written as

$$D_{t+1} = (1+r)D_t - f(K_t) + C_t + I_t^K + I_t^E \tag{2}$$

## 2.2 Standard Modeling Approaches

The dynamic approach to modeling consumption and investment decisions is well known. The normal assumption is that the decision maker allocates resources each period to consumption and investment in productive capital in a way that maximize the present value of utility from consumption. Lower current consumption implies lower current utility but more current investment. This higher investment in turn results in a higher income flow in the future and thus more consumption and more utility in the future. An important distinguishing feature of a life cycle model is the assumption about access to credit for smoothing consumption over time. The standard life cycle model focuses on individual decision making and assumes perfect capital markets. Specifically, the agent borrows and saves freely, constrained only by a lifetime stream of earnings and the restriction that savings/debt must equal zero by the model's termination date. In contrast, the standard optimal growth or Ramsey model was designed to analyze growth of an economy, in which case there is no credit available to smooth consumption over time. This current model has features of a standard life cycle model and a standard optimal growth model because the farmer is assumed to have limited access to credit to smooth consumption and investment in environmental capital over time.

The standard life cycle model has a variety of economic applications including finance, labour and development.[5] The life cycle model has also been used extensively to analyze various aspects of agricultural production, investment and policy [Phimister, 1993, 1995, Boumtje, Barry, and Ellinger, 2001, LaFrance, Pope, and Tack, 2011]. In the simple case of zero capital depreciation and a unit price for the firm's output, an agent with initial debt $D_1$ chooses consumption, $C_t$, and the change in the capital stock (i.e., investment), $K_{t+1} - K_t$, to maximize discounted utility $\sum_{t=1}^{T}(1+\rho)^{-t}U(C_t)$ subject to a lifetime budget constraint, $\sum_{t=1}^{T}(1+r)^{-t}C_t + D_1 \leq \sum_{t=1}^{T}(1+r)^{-t}\left(f(K_t) - (K_{t+1} - K_t)\right)$. Optimized consumption is implied by the Euler equation, $U'(C_{t+1})/U'(C_t) = (1+\rho)/(1+r)$, which ensures that the marginal rate of substitution of consumption across two adjacent time periods is equal to the marginal rate that income can be substituted across two adjacent time periods. Equivalently, $U'(C_t)$ is the marginal utility received from consuming one more unit (i.e., marginal benefit) and it is equal to $(1+r)U'(C_{t+1})/(1+\rho)$, which is the discounted loss in marginal utility from consuming $1+r$ fewer units next period (i.e., marginal opportunity cost). In the special case where $\rho = r$, the Euler equation shows that consumption is constant over time. In the more likely case where $r > \rho$, the assumed concavity of the utility function implies that $C_{t+1} > C_t$ (i.e., consumption is increasing over time).

In the standard life cycle model optimal consumption requires borrowing to finance consumption when cash flow from the farm is comparatively small (i.e., early in the farmer's life cycle) and saving to repay the debt when cash flow from the farm is comparatively large (i.e., later in the farmer's life cycle). The perfect capital markets assumption ensures that the consumption and production decisions are separable. In the absence of investment adjustment costs, borrowed funds are used to finance immediate investment in period one in order to raise the capital stock to its optimal (steady state) level and no additional investment is required beyond period one. In the more realistic case where investment adjustment costs are incorporated into the model there is a gradual transition from the initial capital stock to the steady state capital

[5] See the review article by Blundell [1988] for an overview.

stock. Adjustment costs are not included in the current analysis in order to keep things simple, but this assumption is not critical because growth in productive capital is limited by access to credit.

The simple version of the standard optimal growth model was developed by Ramsey (1928). A good overview of the model is provided by Barrow and Sala-I-Martin [1998]. There are many applications of this model, most in the context of economy wide growth. One particularly nice application of this model is the optimal control of greenhouse gas emissions [Nordhaus, 1992]. The optimal growth model would be an appropriate way to model the problem facing a farm household that operated without access to credit (or with a continuously binding credit limit) so that all consumption and investment had to be financed with current income flows.

In the standard optimal growth model where the time horizon is infinitely long the specific problem (once again with zero depreciation) is to maximize discounted utility $\sum_{t=1}^{\infty}(1+\rho)^{-t}U(C_t)$ subject to a period-by-period budget constraint, $K_{t+1}-K_t = f(K_t)-C_t$. Production is subject to decreasing returns to scale, which implies that the capital stock and consumption both gradually converge to their respective steady-state levels. In the optimal growth model the consumption and investment decisions are obviously not separable. The Euler equation for the optimal growth model can be expressed as $U'(C_{t+1})/U'(C_t) = (1+\rho)/(1+f'(K_t))$. As time progresses $K_t$ gradually increases and $f'(K_t)$ gradually decreases toward $r$ due to ongoing investment. Consequently, the increase in consumption over time is initially large but eventually the consumption path resembles that which emerges in the standard life cycle model. The interpretation of the Euler equation for the optimal growth model is similar to that of the life cycle model except now the opportunity cost of consuming today falls steadily over time due to the declining marginal product of capital.

Constructing a model where a farmer obtains utility from both consumption and the stock of environmental capital would be fairly straight forward for the two types of models discussed above. Unfortunately, neither model is realistic for most agricultural enterprises. The standard life cycle model is not realistic because farmers are typically highly constrained by financing restrictions when contemplating large scale investments such as additional farm land. The standard growth model is also not realistic because farmers typically carry high debt in the early stages of their life cycle in order to benefit from leveraged growth and to smooth consumption over time. A more realistic life cycle model for agriculture should incorporate commonly observed features of debt financing. One such feature is that the farm's debt to asset ratio is normally not allowed by lenders to exceed a pre-determined threshold value (e.g., 50 percent). A second feature is that outstanding debt is normally gradually repaid according to a pre-negotiated schedule after the farm has completed its expansion phase. Phimister [1993] separately incorporates these two features into a standard life cycle model and analyzes the implications of these two constraints. This analysis builds on Phimister's work by assuming that a farm household is restricted by a binding debt to asset ratio during the expansion years of the farm enterprise and repays debt according to a pre-negotiated schedule after the farm has reached a pre-determined maximum size.

Before describing the specific model it is useful to briefly discuss the assumption of decreasing returns to scale. In both the standard life cycle model and the optimal growth model it is necessary to assume decreasing returns to scale to ensure a bounded solution to the problem. The current analysis assumes constant returns to scale primarily to simplify the analysis, but also because for most farming operations it is not realistic to assume decreasing returns to scale for all levels of investment. The most realistic assumption is likely to be constant returns to scale for initial levels of investment and decreasing returns to scale for a sufficiently large level of investment. In the current analysis the farm is constrained by a pre-determined maximum farm size. The value for this parameter will depend on a variety of external variables such as choice of life style, lumpiness in family labour and lack of farm expansion opportunities. With a fixed maximum farm size a model of optimal investment and consumption with constant returns to scale has a well-defined and realistic solution.

## 2.3 Specific Assumptions

The model is divided into a farm expansion phase (hereafter phase 1) and a debt repayment phase (hereafter phase 2). The two phases are separated by a relatively short transition phase that ensures a smooth transition from debt accumulation to debt repayment. During the expansion phase the farmer accumulates productive capital and environmental capital subject to a lending restriction that the farmer's debt to productive capital ratio cannot rise above a threshold value, $\sigma$.[6]. During the debt repayment phase, which is assumed to begin in period $n$, productive capital is held fixed and the goal of the farmer is to repay debt according to a predetermined schedule. With standard concave utility functions and a rate of return on capital that exceeds the cost of borrowing, consumption and investment in environmental capital will rise continually throughout both phases 1 and 2.

The time paths for farm capital, debt, consumption and investment in environmental capital are illustrated in Figure 1. Notice that productive capital rises continually (not necessarily linearly) through out phase 1 and is constant through out the transition phase and phase 2. Debt also rises throughout phase 1 and the transition phase, but it falls and converges toward zero during phase 2. Of particular interest is the stock of environmental capital. The farmer makes an initial investment (or possibly a disinvestment) in period 1 in order to ensure that the stock of environmental capital is on the optimal path. If the desired stock of environmental capital is higher at the beginning of phase 2 than at the end of phase 1 then stock is gradually increased during the transition phase. The transition phase ensures that a decline in consumption is not needed to finance the adjustment in the stock of environmental capital.

Place Figure 1 about here.

Throughout both phases the marginal product of productive capital is assumed to have a constant value, $a$, which implies that the farmer operates with a constant returns to scale production function, $f(K_t) = aK_t$. Assume that $a > r$ to ensure a positive return on investment. The $a > r$ assumption implies that investment in productive capital is always profitable. Consequently, investment will be increased each period to the point where $D_t/K_t = \sigma$ (i.e., the maximum allowable debt to capital ratio is a binding constraint for all periods in phase 1).

[6]Environmental capital is excluded from the farm's asset base for the purpose of calculating the farmer's debt to asset ratio. This is a reasonable assumption if the investment expense is fully sunk or if the lender refuses to include to environmental capital in the calculations because such capital is non-productive.

Phase 1 ends and the transition phase begins when the farm's stock of productive capital reaches level $\hat{K}$. As discussed above this upper limit for productive capital is exogenous to the model. The specific value of $\hat{K}$ will depend on a variety of factors such as the availability of labour and management time, and the overall willingness of the farmer to manage a "large" farm (i.e., a lifestyle choice). By adjusting consumption and investment in environmental capital the farmer can change the rate of investment in productive capital and hence the number of periods in phase 1. Period $n$, which is the starting point of the transition phase of the farmer's life cycle, is endogenous within the model. A description of how the equilibrium value for $n$ is chosen is described below.

In phase 2 the farmer operates with a fixed capital stock, $\hat{K}$, and gradually repays farm debt until time $T$, which for the reasons discussed earlier is assumed to take on a relatively large value. To keep things simple assume that throughout phase 2 outstanding debt is reduced each period by a fixed fraction, $\delta$. In reality debt is typically ammortized, which implies a constant stream of payments over time. The assumption of geometrically declining debt rather than ammortized debt is unlikely to have a significant impact on the general conclusions of this analysis.

In the next section the farmer's investment and consumption decisions are separately analyzed for phase 1 and phase 2. The Euler equations are initially derived and interpreted in general terms. Specific functional forms are later assigned to the utility functions so that closed form solutions can be obtained and numerical simulation results can be generated and presented. After the models for phases 1 and 2 have been fully specified, the transition phase is incorporated in order to link phases 1 and 2.

## 3 Phase 1 Analysis

The formal analysis begins with phase 1, which can be analyzed separate from phase 2 and the transition phase. The phase 1 ending stock of environmental capital will eventually be connected to the beginning stock in the transition phase as a way to link the two components of the model.

The $n-1$ variable that describes the length of phase 1 is initially treated as fixed parameter but eventually an optimal value is chosen.

## 3.1 General Assumptions and Structural Equations

Assume that at the beginning of phase 1 (i.e., beginning of period 1) the level of farm capital is $\hat{K}_1$. As discussed above the assumption of constant returns to scale and $a > r$ implies the farmer will profitably invest at every opportunity and thus the lending ratio, $D_t/K_t \leq \sigma$, will hold as an equality throughout phase 1. It is therefore natural to assume that farm debt at the beginning of period 1, $\hat{D}_1$, is equal to $\sigma\hat{K}_1$ and at the end of period $n-1$, $\hat{D}_n$, is equal to $\sigma\hat{K}$.

Noting that $K_t = D_t/\sigma$ throughout phase 1 and substituting $K_{t+1} - K_t$ for $I_t^K$, it is possible to eliminate $K_t$ from the equation of motion for debt, which is currently given by equation (2):

$$D_{t+1} = (1+r)D_t - \frac{a}{\sigma}D_t + C_t + \frac{1}{\sigma}D_{t+1} - \frac{1}{\sigma}D_t + I_t^E \tag{3}$$

After substituting $E_{t+1} - E_t$ for $I_t^E$, equation (3) can be rearranged and rewritten as

$$D_{t+1} = \frac{(1+\rho)}{\theta_0}D_t - \frac{1}{\theta_1}C_t - \frac{1}{\theta_1}I_t^E \tag{4}$$

where

$$\theta_0 = \left[\frac{1-\sigma}{1+a-(1+r)\sigma}\right](1+\rho) \quad \text{and} \quad \theta_1 = 1/\sigma - 1 \tag{5}$$

It is now possible to determine how the farmer will optimally choose consumption and investment in environmental capital throughout phase 1. The farmer's objective is to maximize the present value of utility from consumption and the stock of environmental capital from period 1 to $n-1$ while facing the budget constraint that is given by equation (4). The boundary conditions for the phase 1 problem are formally specified below. The appropriate Lagrangian equation, with $E_{t+1} - E_t$ substituting for $I_t^E$, can be expressed as

$$L = \sum_{t=1}^{n-1}\left[(1+rho)^{-t}[U(C_t) + V(E_{t+1})] + \lambda_t\left(D_{t+1} - \frac{(1+\rho)}{\theta_0}D_t + \frac{1}{\theta_1}C_t + \frac{1}{\theta_1}E_{t+1} - \frac{1}{\theta_1}E_t\right)\right] \tag{6}$$

The first-order conditions for the optimal choice of $C_t$, $D_{t+1}$ and $E_{t+1}$ for $t = 1, 2, ..., n-1$ can be expressed as

$$\begin{aligned}
&\text{(a) } C_t: \quad (1+\rho)^{-t}U'(C_t) + \frac{1}{\theta_1}\lambda_t = 0 \quad t = 1, 2, ..., n-1 \\
&\text{(b) } D_{t+1}: \quad \lambda_t - \frac{1+\rho}{\theta_0}\lambda_{t+1} = 0 \quad t = 1, 2, ..., n-2 \\
&\text{(c) } E_{t+1}: \quad (1+\rho)^{-t}V'(E_{t+1}) + \frac{1}{\theta_1}\lambda_t - \frac{1}{\theta_1}\lambda_{t+1} = 0 \quad t = 1, 2, ..., n-2 \\
&\text{(d) } \lambda_{t+1}: \quad D_{t+1} = \frac{(1+\rho)}{\theta_0}D_t - \frac{1}{\theta_1}C_t - \frac{1}{\theta_1}(E_{t+1} - E_t) \quad t = 1, 2, ..., n-2
\end{aligned} \tag{7}$$

Equation (7a) can be substituted into equation (7b) to give $U'(C_{t+1}) = \theta_0 U'(C_t)$. Next, substitute equation (7b) into equation (7c) to obtain $(1+\rho)^{-t}V'(E_{t+1}) = (1/\theta_1)[(1+\rho)/\theta_0 - 1]\lambda_{t+1}$. Rewrite this expression for $t-1$ and then use the pair of expressions together with equation (7b) to obtain $V'(E_{t+1}) = \theta_0 V'(E_t)$, which is the same condition as that derived above for optimal consumption. Finally, use equation (7b) to establish $\lambda_t - \lambda_{t+1} = \lambda_t(1-(1+\rho)^{-1}\theta_0)$ and then substitute this latter expression along with equation (7a) into equation (7c) to obtain $V'(E_{t+1}) = (1-(1-\rho)^{-1}\theta_0)U'(C_t)$. In summary, the set of first-order conditions for phase 1 of the farmer's problem can be expressed as

$$\begin{aligned}
&\text{(a)} \quad U'(C_{t+1}) = \theta_0 U'(C_t) \quad t = 1, 2, ..., n-2 \\
&\text{(b)} \quad V'(E_{t+1}) = \theta_0 V'(E_t) \quad t = 1, 2, ..., n-1 \\
&\text{(c)} \quad V'(E_{t+1}) = \left(1 - \frac{\theta_0}{1-\rho}\right)U'(C_t) \quad t = 1, 2, ..., n-1 \\
&\text{(d)} \quad D_{t+1} = \frac{(1+\rho)}{\theta_0}D_t - \frac{1}{\theta_1}C_t - \frac{1}{\theta_1}(E_{t+1} - E_t) \quad t = 1, 2, ..., n-1
\end{aligned} \tag{8}$$

The explicit boundary conditions for the firm's phase 1 problem are $E_1 = \hat{E}_1$, $D_1 = \hat{D}_1 = \sigma\hat{K}_1$ and $D_n = \sigma\hat{K}$. The parameters $\hat{K}_1$ and $\hat{E}_1$ represent the farmer's initial stock of productive capital and environmental capital, respectively. To better understand these restrictions note that the parameter $\hat{K}$ is the maximum amount of capital that the farmer chooses to accumulate and is therefore used to define the end of phase 1 (i.e., $n$ is implied by $K_n = \hat{K}$). As well, the farmer is assumed to begin period 1 and end period $n-1$ with maximum allowable debt, which implies $D_1 = \hat{D}_1 = \sigma\hat{K}_1$ and $D_n = \hat{D}_n = \sigma\hat{K}$.

An immediate complication is that there is no obvious way to utilize the $E_1 = \hat{E}_1$ boundary condition because the optimized series of $E_t$ values only begin in period 2. The reason for this complication is that in period 1 the farmer must make a one-time adjustment investment in environmental capital to ensure that the stock of environmental capital from period 2 onward is on the optimal path. The stock transition is immediate rather than gradual because investment adjustment costs have not been included in the model. In any event, the period one adjustment investment in environmental capital is equal to $E_2 - \hat{E}_1$ where $E_2$ is implied by the solution to equation (8). The $E_2 - \hat{E}_1$ variable, together with the $D_1 = \sigma K_1$ boundary condition, can be substituted into equation (8d) with $t = 2$ in order to establish the following equation.

$$D_2(\hat{D}_1, \hat{E}_1) = \frac{(1+\rho)}{\theta_0}\hat{D}_1 - \frac{1}{\theta_1}C_1 - \frac{1}{\theta_1}(E_2 - \hat{E}_1) \tag{9}$$

The pair of boundary conditions for the firm's stage 1 optimization problem can now be expressed as $D_2 = D_2(\hat{D}_1, \hat{E}_1)$ and $D_n = \sigma\hat{K}$.

## 3.2 Equations for Phase 1 Simulation Model

To keep the analysis simple and to obtain an exact (numerical) solution to the phase 1 problem assume that the pair of utility functions, $U(C_t)$ and $V(E_{t+1})$, are quadratic. Specifically, assume that $U'(C_t) = \alpha_0 - \alpha_1 C_t$ and $V'(E_{t+1}) = \beta_0 - \beta_1 E_{t+1}$. In this special case equation (8b) can be expressed as $E_{t+1} = (\beta_0/\beta_1)(1-\theta_0) + \theta_0 E_t$. The solution to this first order difference equation can written as

$$E_t = \beta_0/\beta_1 + m_1\theta_0^t \tag{10}$$

Within equation (10) $m_1$ is a constant whose value will be determined by the boundary conditions that were specified above.

To obtain an expression for $C_t$ it follows from equation (8c) that $\beta_0 - \beta_1 E_{t+1} = (1 - (1 + \rho)^{-1}\theta_0)(\alpha_0 - \alpha_1 C_t)$. After solving this equation for $C_t$ and substituting in equation (10), the desired expression can be written as

$$C_t = \alpha_0/\alpha_1 + m_1 B\theta_0^t \tag{11}$$

The notation in equation (11) has been simplified by substituting in $B$ where

$$B = \frac{\beta_1}{\alpha_1}\frac{1-\sigma}{a-r\sigma}(1+\rho)$$

.

If equations (10) and (11) are substituted into equation (8d) the state equation for farm debt can be rewritten as

$$D_t = \frac{1+\rho}{\theta_0}D_t - \frac{1}{\theta_1}\frac{\alpha_0}{\alpha_1} - \frac{m_1}{\theta_1}(\theta_0 + B - 1)\theta_0^t \tag{12}$$

The solution to this first order difference equation can be expressed as

$$D_t = m_2\Phi^t - \frac{\alpha_0}{\theta_1\alpha_1}\left(\frac{1-\Phi^t}{1-\Phi}\right) - \frac{m_1(\theta_0 + B - 1)}{\theta_1}\left(\frac{\theta_0^t - \Phi^t}{\theta_0 - \Phi}\right) \tag{13}$$

Note that the parameter $\Phi \equiv (1+\rho)/\theta_0$ has been substituted into equation (10) to simplify the notation.

Equation (10) contains a constant with an unknown value, $m_2$. The boundary conditions that were described above can be used to obtain a value for this constant as well the previously specified $m_1$ constant. Recall that the pair of boundary conditions are $D_2 = D_2(\hat{D}_1, \hat{E}_1)$ and $D_n = \sigma\hat{K}$ To obtain an explicit expression for the right hand side of the first condition substitute equation (10) with $t = 2$ and equation (11) with $t = 1$ into equation (9) to obtain

$$D_2(\hat{D}_1, \hat{E}_1, m_1) = \frac{1+\rho}{\theta_0}\hat{D}_1 - \frac{1}{\theta_1}\left(\frac{\alpha_0}{\alpha_1} + \beta_0/\beta_1 + m_1\theta_0(B+\theta_0) - \hat{E}_1\right) \tag{14}$$

Before equation (14) can be used in the boundary equation the $m_1$ parameter will need to be factored out. Specifically, a new variable, $\hat{D}_2(\hat{D}_1, \hat{E}_1)$ can be defined as

$$\hat{D}_2(\hat{D}_1, \hat{E}_1) = \frac{1+\rho}{\theta_0}\hat{D}_1 - \frac{1}{\theta_1}\left(\frac{\alpha_0}{\alpha_1} + \beta_0/\beta_1 - \hat{E}_1\right) \tag{15}$$

If the factored out term, $-m_1\theta_0(B+\theta_0)/\theta_1$, is subtracted from the right hand side of equation (13) and the resulting expression set equal to equation (15), then the equivalent of the $D_2 = D_2(\hat{D}_1, \hat{E}_1, m_1)$ boundary condition results. The second boundary condition that is relevant for time period $n$ can be constructed by setting the right hand side of equation (13) equal to $\sigma\hat{K}$. The pair of boundary conditions can be expressed in matrix form as

$$\begin{bmatrix} \Phi^2 & -\frac{\theta_0+B-1}{\theta_1}\left(\frac{\theta_0^2-\Phi^2}{\theta_0-\Phi}\right) + \frac{\theta_0}{\theta_1}(B+\theta_0) \\ \Phi^2 & -\frac{\theta_0+B-1}{\theta_1}\left(\frac{\theta_0^n-\Phi^n}{\theta_0-\Phi}\right) \end{bmatrix} \begin{bmatrix} m_2 \\ m_1 \end{bmatrix} = \begin{bmatrix} \hat{D}_2(\hat{D}_1, \hat{E}_1) + \frac{\alpha_0}{\alpha_1\theta_1}\left(\frac{1-\Phi^2}{1-\Phi}\right) \\ \hat{D}_n + \frac{\alpha_0}{\alpha_1\theta_1}\left(\frac{1-\Phi^n}{1-\Phi}\right) \end{bmatrix} \tag{16}$$

The solution values for $m_1$ and $m_2$ in equation (16) can be used in conjunction with equations (10), (11) and (13) to generate a complete solution for phase 1 of the life cycle problem.

### 3.3 Intuition for Phase 1

As discussed above, the model used for this analysis can be interpreted as a hybrid combination of a standard life cycle model and an optimal growth model. It has features of a standard life cycle model because the farmer has some access to credit to smooth consumption and capital investment over time. In also has features of an optimal growth model because capital and production grow over time in proportion to productive investment and this growth in production results in increasing consumption and investment in environmental capital.

To analyze the farmer's choice of investment and consumption, it is useful to take a closer look at the budget constraint and the Euler equations. Recall equation (8d), which shows the state equation for debt with the $D_t/K_t = \sigma$ restriction:

$$D_{t+1} = \frac{(1+\rho)}{\theta_0} D_t - \frac{1}{\theta_1} C_t - \frac{1}{\theta_1}(E_{t+1} - E_t)$$

Substituting $\sigma K_t$ for $D_t$ and noting from equation (5) that $\theta_1 = (1-\sigma)/\sigma$, this equation can be rewritten as

$$K_{t+1} - K_t = \left(\frac{(1+\rho)}{\theta_0} - 1\right) K_t - \frac{1}{1-\sigma} C_t - \frac{1}{1-\sigma}(E_{t+1} - E_t)$$

This new equation shows that the budget constraint for phase 1 is similar to that in the optimal growth model where the price of consumption and investment in environmental capital is equal to $1/(1-\sigma)$ instead of 1.

To better understand the two previous equations let $\Delta_K$ and $\Delta_D$ denote the decrease in investment in productive capital and the decrease in debt, respectively, if consumption or investment in environmental capital increases by one unit. Using the two previous equations and noting that $1/(1-\sigma)$ can be rewritten as $1+\sigma/(1-\sigma)$ it can be seen that $\Delta_K = [1+\sigma/(1-\sigma)]$ and $\Delta_D = \sigma/(1-\sigma)$. These expressions show that increasing current consumption or investment in environmental capital by one unit requires the farmer to lower investment in productive capital by 1 unit plus an additional $\sigma/(1-\sigma)$ units to ensure that the debt to capital ratio remains equal to $\sigma$. The net effect is a decrease in debt equal to $\sigma/(1-\sigma)$ units. Notice that as $\sigma \to 0$ the restriction prevents the farmer from accessing any credit, in which case the the budget constraint for the current model becomes identical to that in the optimal growth model.

Next consider equations (8a) and (8b): $U'(C_t) = \theta_0 U'(C_{t+1})$ and $V'(E_t) = \theta_0 V'(E_{t+1})$. To understand these Euler equations rewrite the expression for $\theta_0$ that is given by equation (5) as $\theta_0 = (1+\rho)/(1+\psi)$ where $\psi = a\Delta_K - r\Delta_D$. By interpreting $\psi$ as the implicit opportunity cost of capital it follows that the Euler equations for optimal consumption and investment in environmental capital are the same as the Euler equation for consumption in the standard life cycle model. It makes sense to interpret $\psi = a\Delta_K - r\Delta_D$ as the opportunity cost of capital because if consumption or investment in environmental capital is reduced by one unit then investment in productive capital will increase by $\Delta_K$ units and this well generate a return next period equal to $a\Delta_K$. Similarly, the one unit reduction will raise net debt by $\Delta_D$ units and this will raise next period's interest expense by $r\Delta_D$ units. Thus, $\psi$ can be interpreted as the return on productive investment minus the cost of financing the investment while accounting for the lender's restriction on the debt to capital ratio and the farmer's desire to maximize leverage.

The pair of Euler equations that are given by equations (8a) and (8b), together with the result that $0 < (1+\rho)/(1+\psi) < 1$, [7] and the assumption that $U$ and $V$ are concave, imply that $C_t$ and $E_t$ are both increasing over time. Moreover, a larger value for $\psi$ implies a more rapid increase for both variables. It should be noted a more rapid increase is equivalent to saying that consumption and investment in environmental capital are more closely matched with the growth in the firm's stock of productive capital, which is a feature consistent with the optimal growth model. A smaller value for $\psi$ means that the farmer makes a larger initial investment in environmental capital and then adds to this capital stock at a slower rate. Such a feature is more reflective of a standard life cycle model where borrowing against the future and smoothing consumption over time is a key objective. The initial investment in environmental capital refers to period 1 adjustment investment, which is illustrated in Figure 1. The relationship between $\psi$ and the direction and size of adjustment investment is too complicated to formally analyze.

[7] The result holds provided that $(1+a)/(1+r) > \sigma$ and $(a-\rho)/(r-\rho) > \sigma$. This pair of conditions hold given the previous assumptions that $0 < \sigma < 1$ and $a > r$.

To better understand the dynamic connection between $C_t$ and $E_t$ consider the final Euler equation, $V'(E_{t+1}) = (1 - (1+\rho)^{-1}\theta_0)U'(C_t)$, which is given by equation (8c). Using the recently defined measure of the farmer's implicit cost of capital, $\psi = a\Delta_K - r\Delta_D$, this Euler equation can be rewritten as

$$U'(C_t) = \frac{1+\psi}{\psi}V'(E_{t+1})$$

This new formulation shows that for each period in phase 1 the farmer should allocate resources in a way that maintains a proportionality between the marginal utility of consumption and the marginal utility of the stock of environmental capital. Notice that the marginal utility of environmental capital is expressed as the present value of an annuity flow. This outcome is expected because environmental capital is a stock variable that generates utility in all future periods as well as the current period whereas consumption is a flow variable that generates utility only in the current period (more details below).

The previous equations can be used to establish a number of comparative static results. First, it should be clear from the $V'(E_t) = (1+\rho)/(1+\psi)V'(E_{t+1})$ Euler equation that a higher value for the farmer's discount rate, $\rho$ reduces the rate of investment in environmental capital. However, although not formally established, it is expected that a higher discount rate will raise period 1 adjustment investment. The fact that a higher discount rate results in higher initial investment and lower subsequent investment implies that the overall relationship between the farmer's discount rate and investment in the environment is ambiguous. In the simulation results presented below the overall relationship turns out to be positive.

Next consider the impact of the $a$, $r$ and $\sigma$ parameters on the rate of investment in environmental capital. It is useful to use two alternative formulations of the implicit opportunity cost of capital: $\psi = a\Delta_K - r\Delta_D$ and $\psi = (a - r\sigma)/(1-\sigma)$. It is obvious from this pair of equations that the farmer's implied opportunity cost of capital, $\psi$, is an increasing function of $a$ and $\sigma$ and a decreasing function of $r$. This result is expected because a higher value for $a$ and a lower value for $r$ means that more revenue is given up if the farmer chooses to allocated an additional dollar to environmental capital. As well, higher $\sigma$ means that each dollar that is not spent on consumption or environmental capital can be more effectively leveraged into productive capital. Based on the previous discussion it therefore follows that higher farm income, more leverage potential and a lower rate of interest on farm debt implies that investment in environmental capital rises more rapidly over time (i.e., is more closely matched with the growth in productive capital). Additional discussion about the connection between these parameters and investment in environmental capital is provided below.[8]

To conclude this section it is useful to revisit the combined Euler equation, $U'(C_t) = [(1 + \psi)/\psi]V'(E_{t+1})$. This equation shows that an increase in the opportunity cost of capital, $\psi$, has a bigger impact on the slope of the $E_t$ function than on the $C_t$ function. In other words, relative to consumption, investing in the environment is more sensitive to changes in the opportunity cost of capital. This outcome emerges because one unit of consumption results in a one unit permanent reduction in future resources whereas investment in environmental capital results in only a temporary reduction.[9]The multiplier $(1 + \psi)/\psi$ shows that the marginal value of investment in environmental capital should be valued as the present value of an annuity flow. It is for this reason that a change in the opportunity cost of capital, $\psi$, affects the marginal incentive to invest in environmental capital more than the marginal incentive to consume.[10]

# 4 Phase 2 and Transition Analysis

The first part of this section lays out the basic assumptions, the general equations and the intuition which describe the optimal path of investment, consumption and farm debt throughout phase 2. Specific functional forms are assigned to the pair of utility functions in the second part of this section in order to generate the simulation results that are presented later in the analysis. The section concludes with a brief explanation of the transition phase.

[8]Because the relationship between $\psi$ and period 1 adjustment investment in environmental capital is too complicated to analyze, it is not possible to comment on the overall extent that phase 1 investment is affected by changes in the various parameters.

[9]In other words at some point in the future the stock of environmental capital can be sold and the proceeds diverted to productive capital.

[10]This situation is similar to why the price of a house is more sensitive to the change in the interest rate than is the rental value of the house.

## 4.1 Structural Equations for Phase 2

Phase 2 begins in period $n + s$ where $n - 1$ is the last period in phase 1 and $s$ is the number of periods in the transition phase (the equilibrium values for $n$ and $s$ are derived below). In phase 2, rather than using rising debt to accumulate capital, the farmer operates with a fixed level of capital, $\hat{K}$, and continually makes payments toward outstanding debt. To keep things simple, assume that the farmer repays a fixed fraction $\delta$ of outstanding debt each period until time $T$ (in other words outstanding debt declines geometrically over time and asymptotically approaches zero). As discussed above $T$ is assumed to take on a large value so that the "end of horizon" effects that are associated with finite length life cycle models with no bequest motive are minimal due to discounting.[11]

The farmer's budget constraint for phase 2 can be constructed as follows. First note that $D_{t+1} = (1 - \delta)D_t$ by assumption. Because productive capital is fixed at $\hat{K}$, it follows that the phase 2 state equation for debt can be expressed as $D_{t+1} = (1 + r)D_t - a\hat{K} + C_t + E_{t+1} - E_t$. Setting this expression equal to $(1 - \delta)D_t$ and substituting $(1 - \delta)^{t-n}\sigma\hat{K}$ for $D_t$ allow the phase 2 budget constraint to be expressed as

$$C_t + E_{t+1} - E_t = a\hat{K} - (r + \delta)D_t \tag{17}$$

The first order conditions for optimally choosing consumption and investment throughout phase 2 are the same as those in phase 1 with $\theta_1 = -1$ except now the budget constraint is given by equation (17) and there is no equation for optimally choosing farm debt. The set of first-order conditions can therefore be written as

$$\begin{aligned}
&\text{(a) } C_t: \quad (1+\rho)^{-t}U'(C_t) = \lambda_t \quad t = n+s, n+s+1, ..., T-1 \\
&\text{(b) } E_{t+1}: \quad (1+\rho)^{-t}V'(E_{t+1}) = \lambda_t - \lambda_{t+1} \quad t = n+s, n+s+1, ..., T-1 \\
&\text{(c) } \lambda_{t+1}: \quad C_t + E_{t+1} - E_t - a\hat{K} + (r+\delta)D_t = 0 \quad t = n+s, n+s+1, ..., T-1
\end{aligned} \tag{18}$$

Equation (18) reveals considerable information about the dynamics of consumption and investment in environmental capital in phase 1. Noting that $\lambda_t$ is a measure of the marginal

[11] A more general model would endogenize the value of $\delta$, possibly by allowing the farmer and the lender to bargain over its value and the corresponding value of $r$.

utility of income, it follows that $\lambda_t$ will drop relatively quickly in the early part of phase 2, which is when debt repayment is largest in absolute terms, and then will slow and approach a steady state value in the latter part of phase 2 (assuming a large value for $T$). Consequently, consumption and investment in environmental capital will increase most rapidly during the early part of phase 2 and then both slow and approach their respective steady state values in the latter part of phase 2.

The economic interpretation of equation (18) is less interesting than in phase 1 because in phase 2 outstanding debt is determined exogenously. The key determinant of investment in environmental capital in phase 2 is the pre-determined rate of debt repayment, which is measured by $\delta$. Phase 2 is nevertheless interesting to include in the model because as is shown below the relationship between consumption and investment in environmental capital is fundamentally different in two phases and so a transition period is required for the farm to adjust the stock of environmental capital from what was optimal in phase 1 to what is optimal in phase 2.

To construct the equations for the simulation model combine the first order conditions to obtain $V'(E_{t+1}) = U'(C_t) - U'(C_{t+1})$. This equation can be combined with the budget constraint that is given by equation (17) to generate a second order difference equation for $E_t$ that describes the path of optimal investment in environmental capital. With quadratic utility where $U'(C_t) = \alpha_0 - \alpha_1 C_t$ and $V'(E_{t+1}) = \beta_0 - \beta_1 E_{t+1}$ the difference equation can be written as

$$E_{t+2} - \frac{2\alpha_1 + \beta_1}{\alpha_1} E_{t+1} + E_t = -\frac{\beta_0}{\alpha_1} + (r+\delta)\delta(1-\delta)^{-n-s}(1-\delta)^t \sigma \hat{K} \tag{19}$$

The solution to this equation is given by

$$E_t = m_3 z_1^t + m_4 z_2^t + \beta_0/\beta_1 + H(1-\delta)^t \tag{20}$$

where

$$z_{1,2} = 1 + \frac{\beta_1}{2\alpha_1} \pm 0.5 \left( \left( 2 + \frac{\beta_1}{\alpha_1} \right)^2 - 4 \right)^{1/2} \quad \text{and} \quad H = \frac{\delta(r+\delta)(1-\delta)^{-n-s}\sigma\hat{K}}{(1-\delta)^2 - (2+\beta_1/\alpha_1)(1-\delta) + 1}$$

With the solution for $E_t$ in hand an expression for optimal consumption can be derived using the budget constraint that is given by equation (17).

Values for the $m_3$ and $m_4$ constants that appear in equation (20) can be determined by using the boundary conditions for the phase 2 problem. The period $T$ boundary condition is given by

$U'(C_{T-1}) = V'(E_T)$. With a quadratic utility function, this condition can be combined with the budget constraint and written as

$$E_T = \Gamma_T + \frac{\alpha_1}{\alpha_1 + \beta_1} E_{T-1} \tag{21}$$

where

$$\Gamma_T = \frac{\alpha_1}{\alpha_1 + \beta_1}\left[a\hat{K} - \left(\frac{\alpha_0 - \beta_0}{\alpha_1}\right)\right] - \frac{\alpha_1(r+\delta)(1-\delta)^{T-n-s-1}}{\alpha_1 + \beta_1}\sigma\hat{K}$$

Similarly, the period $n + s$ boundary condition can be written as $V'(E_{n+s+1}) = U'(C_{n+s}) - U'(C_{n+s+1})$. After substituting in the budget constraint that is given by equation (17) and using the quadratic utility assumption, this boundary condition can be expressed as

$$E_{n+s+1} = \Gamma_{\text{n+s}} + \frac{\alpha_1}{2\alpha_1 + \beta_1} E_{n+s+1} \tag{22}$$

where

$$\Gamma_{\text{n+s}} = \frac{\beta_0 + \alpha_1[\hat{E}_{n+s-1} - \delta(r+\delta)\sigma\hat{K}}{2\alpha_1 + \beta_1}$$

Notice that $\Gamma_{\text{n+s}}$ depends on the ending value of $E_{n+s-1}$ from the transition phase of the farm's life cycle. This is the main connection between these two phases. Values for the constants $m_1$ and $m_2$ can be obtained by substituting equation (20) first with $t = T$ and then with $t = T - 1$ into equation (21) and by substituting equation (20) first with $t = n + s + 1$ and then with $t = n + s + 2$ into equation (22). The resulting set of equations can be solved to provide equilibrium values for $m_3$ and $m_4$.

## 4.2 Transition Phase

When the farmer finishes phase 1 the debt to equity ratio is at a maximum allowable level. Consequently there is no additional credit available for the farmer to adjust the inventory of environmental capital upward before beginning phase 2 if such an adjustment is needed. A secondary problem is that the farmer may initially have to decrease consumption in order to finance the loan repayment. To address this issue it is reasonable to assume a transition phase that separates phases 1 and 2. To keep things simple assume that throughout the transition phase the farmer operates with capital fixed at $\hat{K}$, debt fixed at $\sigma\hat{K}$ and consumption fixed at

$C^*_{n-1}$ (i.e., the level that was optimal for the final period of phase 1). All residual cash from farming during the transition phase are allocated to building up the stock of environmental capital. The transition phase continues until the stock of environmental capital has reached a level that is optimal in period $n + s + 1$. This strategy eliminates the need for large increases in investment in environmental capital and a corresponding large decrease in consumption within a single period. The length of the transition phase that achieves this outcome is what defines the equilibrium value of $s$.[12]

The transition phase begins in period $n$. The farmer will choose $n$ to maximize the present value of utility that is earned in all three phases. The farmer does not want to choose an excessively low value for $n$ because that would mean forfeiting an excessive amount of consumption and investment in environmental capital due to the heavy investment requirements. Conversely the farmer does not want to choose and excessively large value for $n$ because that would imply a relatively slow increase in the build up of productive capital, all of which generates a positive return for the farmer. In the simulation modeling it turns out that with most parameter values the present value of utility is maximized by continuing to increase the value of $n$ until $K_t$ begins to rise above $\hat{K}$ at some point in phase 1. In most cases $n$ is approximately defined as the value of $n$ which solves $K_n = \hat{K}$.

# 5 Simulation Results

The previous equations were programmed in Microsoft Excel to simulate optimal consumption and investment in the life cycle model for $T = 100$. Table 1 shows the values that have been assigned to the various parameters of the simulation model. Excluding the utility function parameters, the assigned values are intended to be realistic. For example, the base case equilibrium length of phase 1 is $n = 16$ (details below), which can be interpreted as 16 years. The assumed parameter values in Table 1 therefore reveal that the farm grows from 10 units to 75 units (a factor of 7.5) over a 16 year period. The starting stock of environmental capital is -10, which implies that the farm begins operations at 10 units below minimum recommended environmen-

[12]Because $s$ must be an integer, fraction values for the calculated value of $s$ are rounded up.

| Description | Symbol | Value | Description | Symbol | Value |
|---|---|---|---|---|---|
| Maximum $K_t$ | $\hat{K}$ | 75 | Rate of Discount | $\rho$ | 0.05 |
| Starting $K_t$ | $\hat{K}_1$ | 10 | Debt Repayment Fraction | $\delta$ | 0.075 |
| Starting $E_t$ | $\hat{E}_1$ | -10 | Intercept of $U'(C_t)$ | $\alpha_0$ | 10 |
| Maximum $D_t/K_t$ | $\sigma$ | 0.5 | Slope of $U'(C_t)$ | $\alpha_1$ | 1 |
| MP of Capital | $a$ | 0.08 | Intercept of $V'(C_t)$ | $\beta_0$ | 0.5 |
| Cost of Capital | $r$ | 0.05 | Slope of $V'(C_t)$ | $\beta_1$ | 0.1 |

Table 1: Base Case Parameter Values

tal capital. The maximum allowable debt to capital ratio is $\sigma = 0.5$, which implies that the lender requires one unit of secured capital for every unit of non-secured capital. Farm capital is assumed to generate a constant return of 8 percent per year ($a = 8$), the rate of interest on borrowed funds is 5 percent ($r = 0.05$) and the farmer's rate of discount is 5 percent ($\rho = 0.05$). The farmer is required to repay $\delta = 7.5$ percent of outstanding debt each period. No attempt is made to justify the choice of parameter values for the pair of utility functions. Because there is no uncertainty in the model any concave function are expected to generate results that are qualitatively similar.

The base case results are illustrated in Figure 2. Only the first 40 years of the farmer's $T = 100$ period time horizon are shown on the horizontal axis. The left hand vertical axis measures the stock of productive capital and outstanding farm debt. The right hand vertical axis measures the level of consumption and the stock of environmental capital. For the base case the equilibrium value of $n$ is 16 years and the equilibrium value of $s$ is 3 years.

The various schedules in Figure 2 have the expected shapes. In phase 2 as the age of the farmer increases the level of outstanding debt declines toward zero and both the level of consumption and investment in environmental capital approach their respective steady state levels. The period 1 stock of environmental capital was purposely chosen to preclude the need for an adjustment investment in period 1. Notice that the stock of environmental capital is negative (i.e., below the recommended minimum standard) until a few years into phase 2. This implies that the farm is meeting environmental standards only after debt is starting to be repaid. Consumption is also negative until about year eight, which implies that the farm is relaying on off-farm income to finance both consumption and part of the investment in productive and environmental capital. The three period transition phase is sufficient to ensure that the stock of environmental capital transitions smoothly between phases 1 and 2.

Regarding Figure 2 it is important to explain the reason for the large decline in consumption that occurs at the beginning of phase 2. It is best to view consumption as being above "average" in phase 1 because investing in environmental capital is relatively unattractive throughout phase 1. In phase 2 when consuming and investing in environmental capital have similar appeal the farmer reduces consumption and raises investment in environmental capital relative to the phase 1 time paths. The reason why consuming in phase 1 is more appealing than investing in environmental capital is because the opportunity cost of diverting resources away from productive investment is relatively high in phase 1 due to leverage. As discussed above, investment is more sensitive to a higher opportunity cost because investment impacts a potential flow of resources across many periods whereas consumption impacts a potential flow for just the current period. It is this annuity feature of the investment that makes it more sensitive to the cost of capital.

Place Figure 2 about here.

The sensitivity results that are displayed in Table 2 show how the key variables are impacted by changes in the key exogenous parameters. The first column shows the equilibrium value for $n$ and the next three columns show the stock of environmental capital in period 2, in period $n$ immediately before the transition phase begins, and in period $n + s$, immediately after the completion of the transition phase. The last two columns, respectively, are present value measures of the stock of environmental capital and consumption across all periods. In each row the results of the variables being examined are displayed both before (base case) and after the indicated change in the exogenous parameter.

The first two rows of results in Table 2 reveal that an increase in the interest rate from $r = 0.05$ to $r = 0.06$ results in an additional two years of farm expansion. This result is expected because a higher interest rate lowers the farmer's opportunity cost of not leveraging investment in productive capital by instead allocating resources to consumption and environmental capital.

Table 2 shows that the present value of consumption has declined due to the higher cost of financing interest expense rather than consumption. More surprising, however, is the result that the present value of the stock of environmental capital is higher due to the increase in the interest rate. The higher present value can be attributed to the fact that with higher $r$ the farmer invests more aggressively in phase 1 and less aggressively in phase 2, but with discounting the phase 1 outcomes receive more weight. As discussed above the differential in the opportunity cost of diverting resources away from productive capital induces the farmer to invest in environmental capital relatively less heavily during phase 1. Consequently, an increase in $r$ which lowers the farmer's opportunity cost of diverting resources away from productive capital offsets part of that differential.

Place Table 2 about here.

The second pair of rows in Table 2 reveal the impact of an increase in the farmer's discount rate from $\rho = 0.05$ to $\rho = 0.06$. Not surprisingly the present value of both consumption and the stock of environmental capital are lower because of the higher discount rate. More importantly, notice that this change has induced the farmer to remain in phase 1 for 19 years rather than 16 years. Moreover, the stock of environmental capital is higher throughout both phases so that in non-discounted terms the higher discount rate has raised investment in environmental capital. This outcome is expected because with a higher discount rate the opportunity cost of investing in environmental capital rather than productive capital is lower because future income flows that result from investment in productive capital are discounted more heavily. Consequently, the farmer will invest more aggressively in environmental capital and will remain in phase 1 longer.

In the third pair of rows in Table 2 the analysis involves raising the return on productive capital from 0.08 to 0.09. This change results in a substantial increase in the present value of both consumption and the stock of environmental capital. The impact on $n$ is slightly negative. Of particular interest is the connection between $a$ and investment in environmental capital because an increase in $a$ could be the result of a new farm income stabilization program. The increase in $a$ lowers investment in environmental capital in phase 1 and raises it during phase 2. However, due to discounting the loss more than offsets the gain so the overall effect is a decrease in the present value of the stock of environmental capital. The increase in $a$ reduces investment in phase 1 because it raises the opportunity cost of not diverting resources into leveraged investment in productive capital. Conversely, the decrease in $a$ increases investment in phase 2 because more residual cash is available after farm debt has been serviced, so the amount allocated to environmental capital increases.

The fourth pair of rows in Table 2 show how the results are impacted by an approximate 12 percent increase in maximum farm size, $\hat{K}$. Not surprisingly the length of phase 1 increases with this change, as well as the present value of consumption. This is because with a larger farm and the assumption of constant returns to scale and a positive return on capital investment, it must be the case that more overall income is generated on a larger farm. However, similar to the case of higher farm income, an increase in maximum farm size induces the farmer to invest less in environmental capital. The reason is that a larger maximum farm size raises the opportunity cost of diverting farm resources away from productive investment during phase 1, and even though the additional income raises investment in environmental capital in phase 2 the overall impact of a larger farm size is negative.

The fifth pair of rows in Table 2 show the impact of a lower period 1 starting value for environmental capital. Notice that the effect of the lower starting value has been more than fully offset by the time the farmer starts phase 2. Despite the elimination of this loss, however, the present value of the stock of environmental capital is significantly, lower due to the lower value of $E_1$. The higher level of investment in environmental capital that is required to eliminate the starting value effects also results in a lower present value of consumption.

The third pair of rows from the bottom in Table 2 show the impact of increasing the maximum allowable debt to asset ratio from 0.5 to 0.6. This change raises the present value of both consumption and investment because it allows the farmer to leverage farm income more effectively in phase 1. Notice that maximum farm size is now reached after 12 years instead of 16 years. During phase 1 the higher value of $\sigma$ raises the opportunity cost of investing in environmental capital and so this level of investment falls below the base case. However, phase 1 is relatively short and there is a large upward "jump" in the stock of environmental capital in period 12. The net result is that the present value of the stock of environmental capital is higher due to higher leverage.

The bottom row of Table 2 shows the results for the case where the farmer is required to repay 10 percent of outstanding debt each period rather than 7.5 percent. This change has no impact on the phase 1 outcomes, including $n$. The higher value for $\delta$ raises the shadow cost of debt in phase 2 which in turn reduces the farmer's incentive to invest in environmental capital throughout phase 2. Consequently, the present value of aggregate investment in phase 2 is lower due to the less liberal repayment terms that is set by the lender.

Finally, the last row in Table 2 examines the effect of additional weight on environmental outcomes in the farmer's utility function. Not surprisingly, this shift increases overall investment in environmental capital at the expense of lower consumption. The value of $n$ has also increased by one year relative to the base case, which can be explained by noting that farm income is being diverted away from productive capital and toward environmental capital.

# 6 Concluding Comments

The purpose of this paper was to develop a life cycle model of farm consumption and investment in both productive and environmental capital and then use the model to examine the determinants of environmental investments. Rather than using a standard life cycle model within which credit markers are assumed to be perfect or using an optimal growth model within which there is no credit, the approach here is to assume leveraged investment subject to an exogenous lending constraint for phase 1 and debt repayment subject to an exogenous repayment schedule for phase 2. The model relies on an exogenous restriction on maximum farm size rather than on implicitly rising cost of expansion due to a decreasing returns to scale production technology. Although this assumption reduces the generality of the results, the assumption of an exogenous maximum and constant returns to scale greatly simplified the analysis and gave rise to clear predictions and economic intuition for a relatively large number of comparative static outcomes. The assumption of quadratic utility that forms the basis of the simulation model likely diminished the generality of the empirical results. Nevertheless, in a model without uncertainty the gain from working with a more general utility function is unclear.

There are many possible extensions for this model. For example, it would be desirable to include a bequest motive at time $T$ and then examine the effects of different lengths of time horizon on the incentive to invest in environmental capital. Another desirable extension would be to add farm income uncertainty to determine if uncertainty affects investment in environmental capital differently than how it affects investment in productive capital. Third, the model could be generalized in an interesting and useful way if investment in environmental capital had a specific impact on the productive capacity of the farm. Finally, it would be desirable to allow for an interaction between the marginal utility of consumption and the marginal utility of the stock of environmental capital. Each of these proposed extensions would make the results of the model more general and give rise to new ways of thinking about how farmer's view investment in environmental capital.

Finally, this analysis is a good starting point for policy analysis because it provides policy makers with a much broader framework for thinking about farm level response to environmental programs. Policy makers should think about how the incentives to invest change over the course of farmer's life cycle, how restrictions on access to credit and repayment terms are important determinants of the incentive to and perhaps most importantly how an increase in farm income gives rise to offsetting incentives for a farmer to add to the stock of environmental capital. The model is not yet well enough developed to accommodate the inclusion of specific environmental programs, but moving in that direction certainly appears warranted.

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| | n | $E_2$ | $E_n$ | $E_{\text{n+s}}$ | $PV(E_t)$ | $PV(C_t)$ |
|---|---|---|---|---|---|---|
| Base | 16 | -9.171 | -1.509 | -0.122 | -35.409 | 204.53 |
| $\Delta r$ = 0.01 | 18 | -7.771 | -1.067 | -0.220 | -30.162 | 165.03 |
| Base | 16 | -9.171 | -1.509 | -0.122 | -35.409 | 204.53 |
| $\Delta\rho$ = 0.01 | 19 | -8.552 | -1.190 | -0.075 | -38.893 | 120.82 |
| Base | 16 | -9.171 | -1.509 | -0.122 | -35.409 | 204.53 |
| $\Delta a$ = 0.01 | 15 | -11.803 | -1.469 | -0.473 | -47.156 | 302.13 |
| Base | 16 | -9.171 | -1.509 | -0.122 | -35.409 | 204.53 |
| $\Delta\hat{K}$ = 10 | 18 | -9.314 | -0.883 | 0.340 | -37.662 | 220.32 |
| Base | 16 | -9.171 | -1.509 | -0.122 | -35.409 | 204.53 |
| $\Delta E_1$ = -5 | 17 | -9.886 | -1.468 | -0.164 | -50.322 | 141.99 |
| Base | 16 | -9.171 | -1.509 | -0.122 | -35.409 | 204.53 |
| $\Delta\sigma$ = 0.1 | 12 | -11.022 | -3.037 | -1.070 | -41.274 | 218.03 |
| Base | 16 | -9.171 | -1.509 | -0.122 | -35.409 | 204.53 |
| $\Delta\delta$ = 0.025 | 16 | -9.171 | -1.509 | -0.816 | -36.065 | 200.13 |
| Base | 16 | -9.171 | -1.509 | -0.122 | -35.409 | 204.53 |
| $\Delta\beta_0$ = 0.25 | 16 | -7.033 | 0.825 | 1.686 | -10.972 | 171.66 |

Table 2: Sensitivity Results

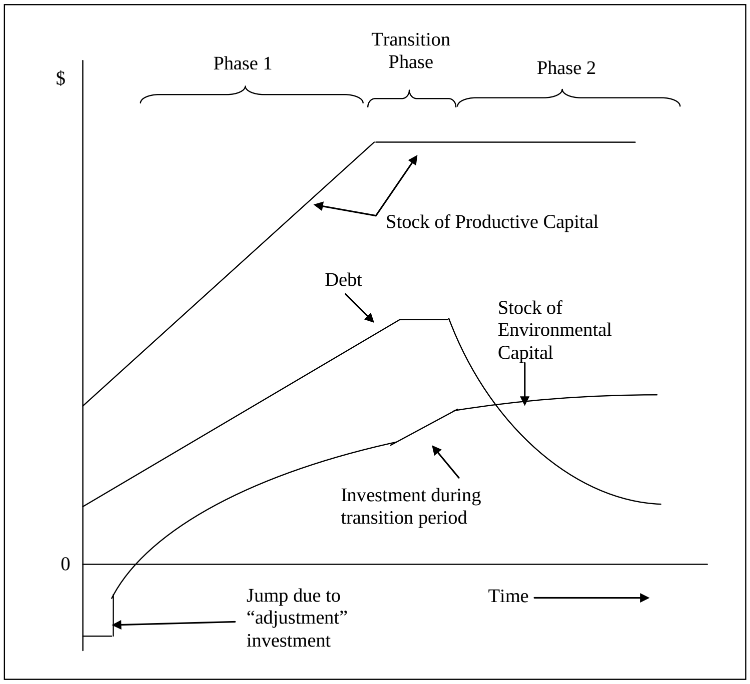


Figure 1: Model Overview

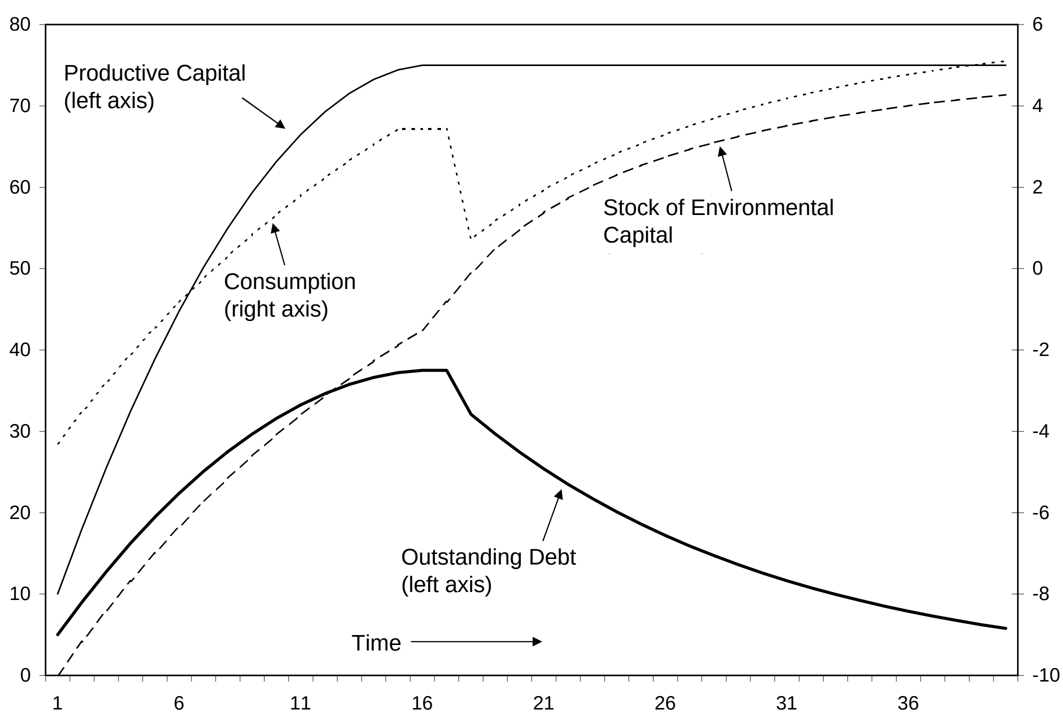


Figure 2: Base Case Simulation Results